

Volume
One

Statistical
Tools



Practical Tools for Continuous Improvement

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WHAT IS IT?

Capability analysis is a set of calculations used to assess whether a system is statistically able to meet a set of specifications or requirements. To complete the calculations, a set of data is required, usually generated by a ***control chart***¹; however, data can be collected specifically for this purpose.

Specifications or requirements are the numerical values within which the system is expected to operate, that is, the minimum and maximum acceptable values. Occasionally there is only one limit, a maximum or minimum. Customers, engineers, or managers usually set specifications. Specifications are numerical requirements, goals, aims, or standards. It is important to remember that specifications are not the same as control limits. Control limits come from control charts and are based on the data. Specifications are the numerical requirements of the system.

Traditionally, capability analysis has been utilized primarily by manufacturing organizations. However, capability is now being applied to service and administration activities. All industries have numerical requirements and can collect data from systems. Capability analysis can therefore be useful in any industry.

There are various methods for analyzing capability, depending on the type of data, how it is collected, and the characteristics of the data. The standard or most commonly used method is shown first. Other approaches are shown in “Variations” later in this section.

All methods of capability analysis require that the data is statistically stable, with no special causes of variation present. To assess whether the data is statistically stable, a control chart should be completed. If special causes exist, data from the system will be changing. If capability analysis is performed, it will show approximately what happened in the past, but cannot be used to predict capability in the future. It will provide only a snapshot of the process at best. If, however, a system is stable, capability analysis shows not only the ability of the system in the past, but also, if the system remains stable, predicts the future performance of the system.

Capability analysis is summarized in indices; these indices show a system’s ability to meet its numerical requirements. They can be monitored and reported over time to show how a system is changing. Various capability indices are presented in this section; however, the main indices used are C_p and C_{pk} . The indices are easy to interpret; for example, a C_{pk} of more than one indicates that the system is producing within the specifications or requirements. If the C_{pk} is less than one, the system is producing data outside the specifications or

¹ Items that are in bold italics refer to other tools in this book.

requirements. This section contains detailed explanations of various capability indices and their interpretation.

Capability analysis is an excellent tool to demonstrate the extent of an improvement made to a process. It can summarize a great deal of information simply, showing the capability of a process, the extent of improvement needed, and later the extent of the improvement achieved.

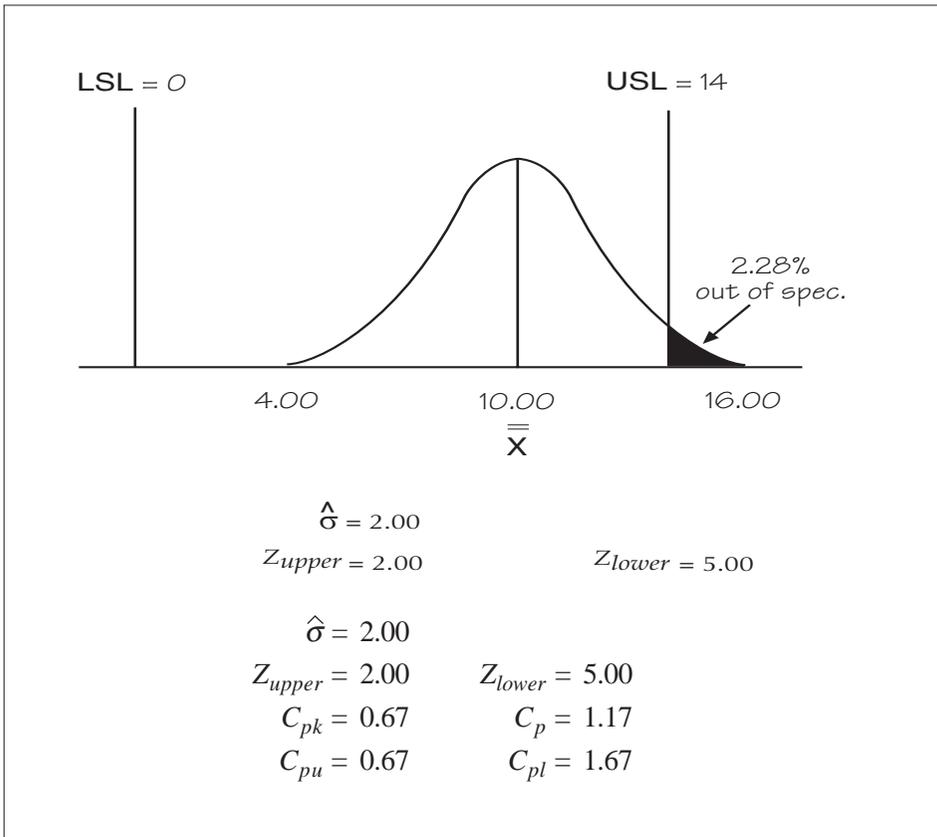
Capability indices help to change the focus from only meeting requirements to continuous improvement of the process. Traditionally, the focus has been to reduce the proportion of product or service that does not meet specifications, using measures such as percentage of nonconforming product. Capability indices help to reduce the variation relative to the specifications or requirements, achieving increasingly higher C_p and C_{pk} values.

Before capability analysis is completed, a *histogram* and *control chart* need to be completed. This section assumes that these tools have been studied previously.



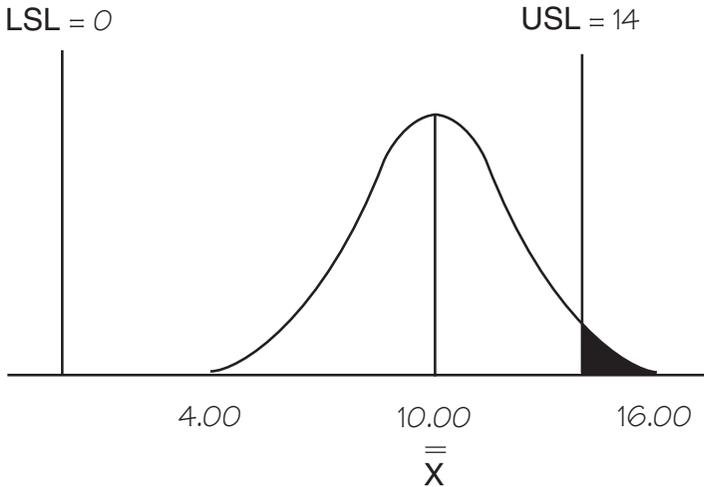
WHAT DOES IT LOOK LIKE?

A courier company has set up a team to look at the actual arrival time at customers' locations to pick up packages, in comparison to the scheduled arrival time. The company guarantees pick up of packages within 14 minutes of the scheduled time. It is unacceptable to customers for the courier to arrive early. Therefore, there are two requirements: on time and up to 14 minutes late. The result of the capability analysis for this example follows.



4. DRAW THE SPECIFICATION LIMITS ON THE DISTRIBUTION.

Draw vertical lines on the distribution to represent the lower and upper specification limits. In the example, the lower specification limit (LSL) is 0 minutes (on time) and the upper specification limit (USL) is 14 minutes. Estimate where the two lines should be located in reference to the overall average and the tails of the curve. Label each specification with its abbreviation and value. The example completed through this step follows.



The diagram shows whether any portion of the curve is beyond the specifications. In the example, some of the distribution is beyond the upper specification. If the overall average of the distribution is outside the specification, refer to “Variation – Capability analysis where the overall average is outside the specification” later in this section.

5. CALCULATE HOW MUCH DATA IS OUTSIDE THE SPECIFICATIONS.

As indicated in the previous step, some of the distribution is outside the specification limit. The question is, how much? To determine the percentage that falls outside the specification limits, it is necessary to find how many estimated standard deviations exist between the overall average and each specification limit. The number of standard deviations is known as the Z value. Z values are used to determine the percentage of output that is outside the specification limits using the “Standard normal distribution table.”

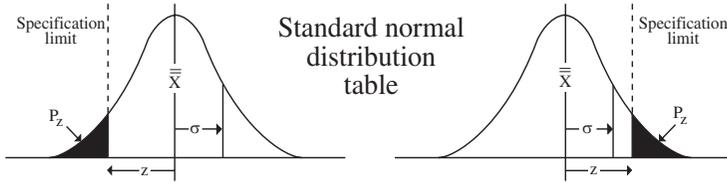
a. Find the percentage above the upper specification.

The first step in determining the percentage above the upper specification is to calculate the Z value for the upper specification. This is found by subtracting the overall average from the upper specification, and then dividing by the estimated standard deviation. The Z value for the upper specification is denoted as Z_{upper} . The upper specification for the example is 14, the overall average is 10.00, and the estimated standard deviation is 2.00. Thus, the value of Z_{upper} for the example is:

$$\begin{aligned} Z_{upper} &= \frac{USL - \bar{X}}{\hat{\sigma}} \\ &= \frac{14.00 - 10.00}{2.00} \\ &= 2.00 \end{aligned}$$

This means that the upper specification is located 2.00 estimated standard deviations away from the overall average. Look up the Z value in the “Standard normal distribution table” to find the estimated proportion of output that is outside the upper specification. The table follows on the next page.

Z values are listed along the left and top of the table. The whole number (number to the left of the decimal) and the tenths digit (first number to the right of the decimal) are listed on the left hand side of the table, and the hundredths digit (second number to the right of the decimal) is along the top. The table shows Z values only up to 4. If the Z value is greater than 4, the proportion outside the specification is virtually 0. In the example, the Z value is 2.00. To find the percentage outside the specification, go down the left hand side of the table to 2.0 and then across to the column marked x.x0. The number is 0.0228, which is the proportion outside the specification. To convert the proportion to a percentage, multiply it by 100. The percentage outside the upper specification is 2.28 percent. Place this percentage on the diagram.



z	x.x0	x.x1	x.x2	x.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
4.0	.00003									
3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

b. Find the percentage below the lower specification.

The Z value for the lower specification is found by subtracting the lower specification from the overall average, and then dividing by the estimated standard deviation. The Z value for the lower specification is denoted as Z_{lower} . The lower specification for the example is 0, the overall average is 10.00, and the estimated standard deviation is 2.00. Thus, the value for Z_{lower} for the example is:

$$\begin{aligned} Z_{lower} &= \frac{\bar{\bar{X}} - LSL}{\hat{\sigma}} \\ &= \frac{10.00 - 0.00}{2.00} \\ &= 5.00 \end{aligned}$$

This means that the lower specification is located 5.00 estimated standard deviations away from the overall average. Look up the Z value in the “Standard normal distribution table” as previously described. Since the table shows Z values up to only 4, the proportion and percentage outside of the specification is taken as 0. If any of the data is outside the specification, add the percentage to the diagram.

c. Determine the total percentage outside the specifications.

The total percentage outside the specification limits or requirements is found by adding the percentage outside the upper and lower specification limits. The total percent of output located outside the specification limits for the example is:

$$2.28 + 0 = 2.28\%$$

6. CALCULATE AND INTERPRET THE CAPABILITY INDICES.

This step describes the key capability indices. Other indices are described in “Getting the most from capability analysis,” later in this part.

a. Calculate Cp.

Cp is an index used to assess the width of the process spread in comparison to the width of the specification. It is calculated by dividing the allowable spread by the actual spread. The allowable spread is the difference between the upper and lower specification limits. The actual spread is 6 times the estimated standard deviation. Plus or minus 3 times the estimated standard deviation contains 99.73 percent of the data and is commonly used to describe actual spread.